Hans Cloos Lecture for 14th IAEG Congress

STATISTICAL MECHANICS OF ROCK MASSES (SMRM)

---- An Engineering Geology based Rock Mechanics

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Thanks to IAEG for recognizing "Statistical Mechanics of Rock Masses" and award the "Hans Cloos Medal"!

Thanks to China National Natural Science Fundation, all the institutions and colleagues who support and help the development of "SMRM" system!

Contents：

- The world needs engineering geo-mechanics!
- What does SMRM do?
- Applied Technology of SMRM?

I. THE WORLD NEEDS ENGINEERING GEO-MECHANICS!

- **EG Ensures Construction & Security of the World !**
  - More than half of infrastructure constructions are on/in rock masses.

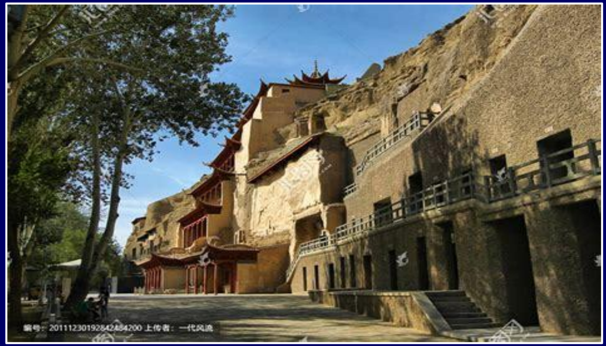

Super hydropower station


Railway tunnels


Large mines


Bridges in mountain


Large radio telescope


Dunhuang rock cave

- Lots of geo-hazards due to inadequate Eng-geo-mechanical understanding & techniques.

Highway Landslide

Highway Rockfall

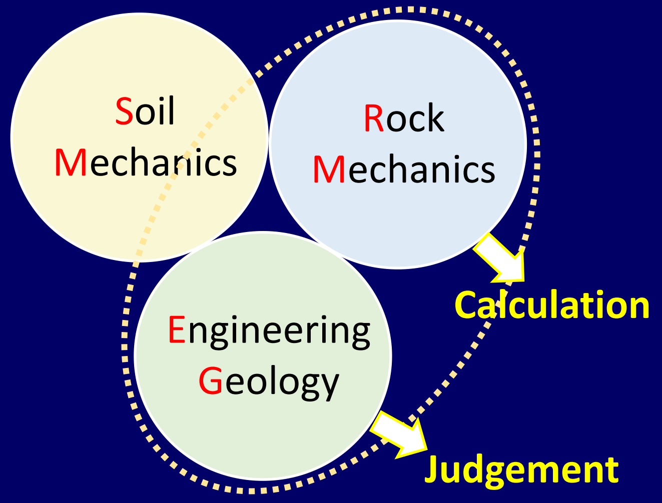
Mine Prop Failure

Tunnel Roof Collapse

Tunnel Water Inrush

Tunnel Lining Failure

- # Challenge to Rock Engineering Geology!



The Iron Triangle:
Rely on each other;
Support each other!

- Engineering Geology mainly do geological judgement but less mechanical analysis.
- Rock Mechanics is good at mechanical analysis, but short to evaluate Engineering Geological problems.
- **The world needs Engineering Geology based rock mechanics!** This is why H. Cloos, L. Müller and DZ Gu founded "Eng-Geo-Mechanics".

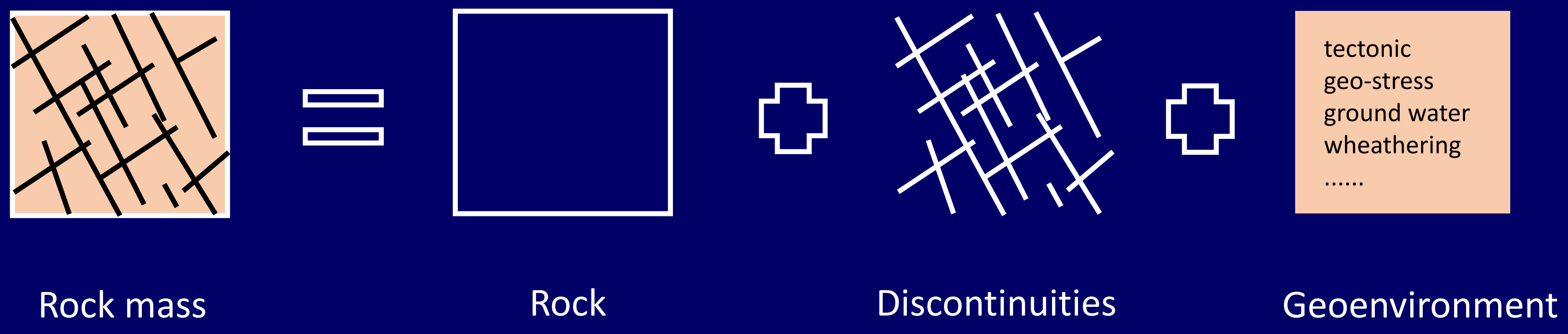
II. WHAT DOES SMRM DO?

SMRM: STATISTICAL MECHANICS OF ROCK MASSES

---- A Rock Mechanics from Engineering Geology

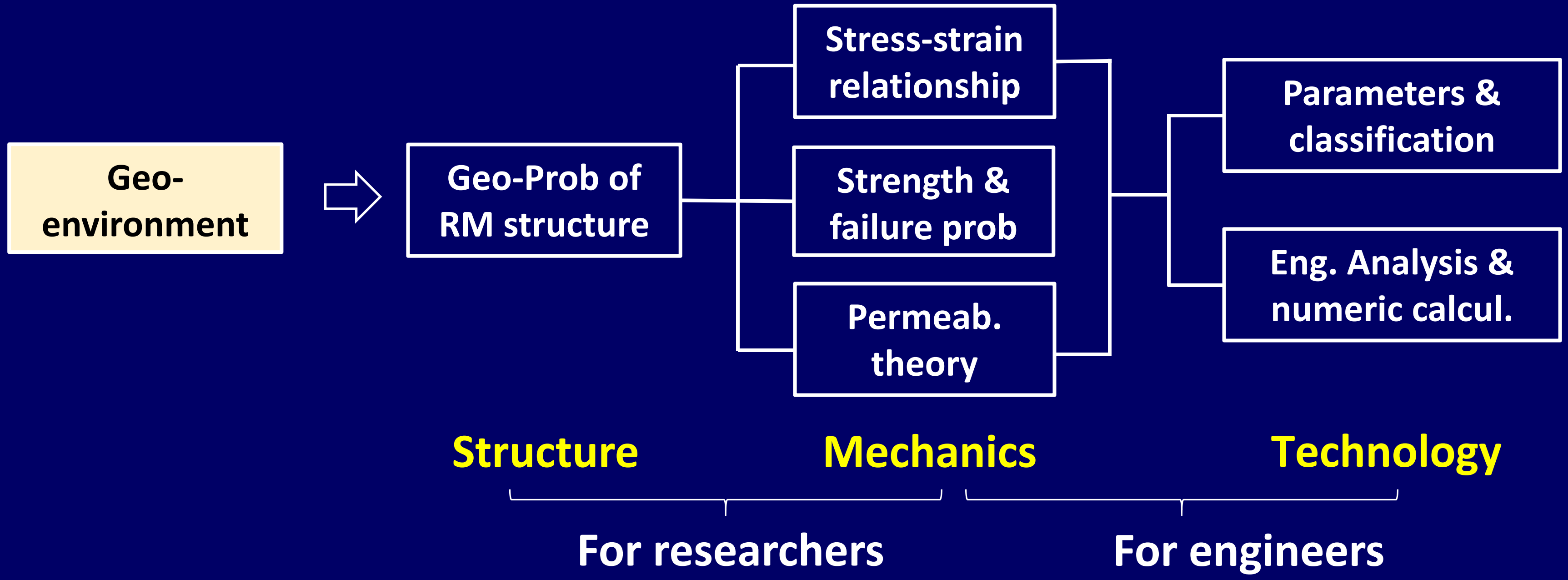
1. Basic understanding to rock masses

- A mass of rock and discontinuity network under certain geo - environment;
- Mechanical beheavior controlled by rock, joints and geo-environmental factors.



2. Ideology of SMRM

- **GEOLOGICAL BASIS:** Investigate the mechanical behavior of rock masses under geo-environment;
- **STATISTICAL PHYSICS:** Find macro regulation from micro behavior of rock & joints.



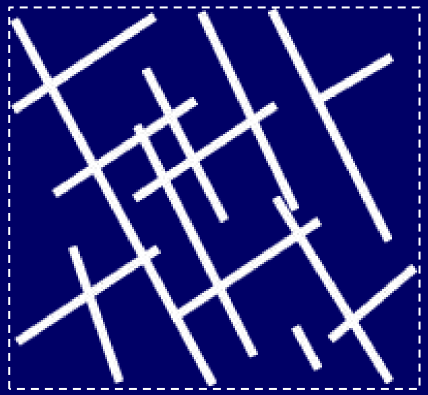
3. Developing History of SMRM

- Before 1980s, mechanics of rock and joint;
- Hudson, Priest *et al* (1980s), *:* geometric probability of discontinuities;
- Oda (1983), fabric tensor based mechanical model for rock mass;
- Kawamato (1988), damage mechanical model of rock mass;
- Faquan Wu (1993), “Principles of Statistical Mechanics of Rock Mass”, developing SMRM technology, and
- “Statistical Mechanics of Rock Mass--Theory and Application” with “SMRM Calculation” system (2022).

4. Four basic models for SMRM

(1) Geometric probability models of rock mass structure

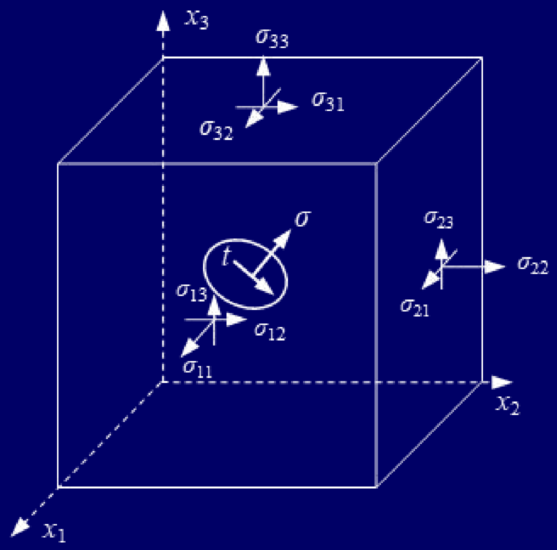
- Basic Data: Joint set No., Attitude, Density, Radius, Opening …



- Volume density $\lambda_v = \frac{2}{\pi^3}\sum^{m} \quad \mu^2\lambda = \frac{1}{2\pi}\sum^{m}\frac{\lambda}{\bar{a}^2}$
- *RQD* $RQD = \frac{1}{1-e^{-\lambda L}}(e^{-\lambda t} - e^{-\lambda L}) \times 100\%$
- Max. radius $a_m = \bar{a}\ln(\lambda_v \cdot V) = \bar{a}\ln(\frac{\lambda}{2\pi\bar{a}^2}V)$
- Volume connect. $\eta_{ji} = \pi\lambda_{vj}\bar{t}_j \ (\bar{a}_j + r_j) \ ^2 e^{-\frac{3r_j}{\bar{a}_j}}$

（2） SMRM stress-strain model

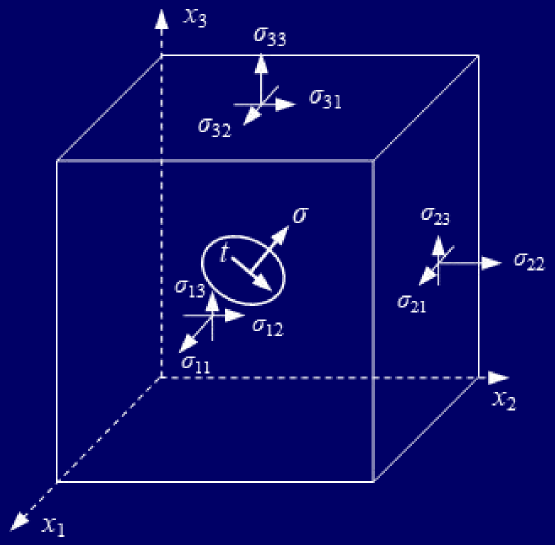
- Strain energy density model considering major geo-factors;
- ➩ Elastic stress-strain relatoinship.



- strain energy density $u = u_0 + \sum_{i=1} u_{ci}$
- stress-strain relation $e_{ij} = (C_{0ijst} + C_{cijst})\sigma_{st}$
- flexibility tensor for rock $C_{0ijst} = \frac{1+v}{2E}(\delta_{is}\delta_{jt} + \delta_{it}\delta_{js}) - \frac{v}{E}\delta_{ij}\delta_{st}$
- flexibility tensor for joints $C_{cijst} = \frac{\alpha}{E}\sum^{m} \lambda\bar{a}\,[k^2 n_i n_t + \beta h^2(\delta_{it} - n_i n_t)]n_j n_s$

(3) SMRM strength & Failure probability

- Jopint fracture mechanical criterion $K_{Ic}^2 = \frac{4}{\pi} a_m (k^2\sigma^2 + \beta\tau^2)$
- weak link hypothesis



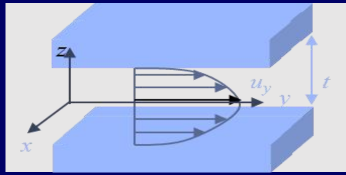
- weak link strength $\sigma_{1c} = \min(\sigma_{cr},\ \sigma_{cj},\ i = 1,2,\ldots m)$

$$\begin{cases} \sigma_{cr} = \sigma_3 \tan^2\theta + R_c, & \theta = 45^\circ + \frac{\varphi_r}{2} \\ \sigma_{cj} = \frac{-B - \sqrt{B^2 - 4AC}}{2A}, & \begin{cases} A = [(1+f^2)n_3^2 - 1]n_3^2 \\ B = 2[af + (1+f^2)\sigma]n_3^2 \\ C = a^2 + 2af_j\sigma + (1+f^2)\sigma^2 - p^2 \\ a = \frac{K_{Ic}}{2}(\frac{\pi}{\beta a_m})^{1/2} + c \\ \sigma = n_1^2\sigma_x + n_2^2\sigma_y \\ p^2 = n_1^2\sigma_x^2 + n_2^2\sigma_y^2 \end{cases} \end{cases}$$

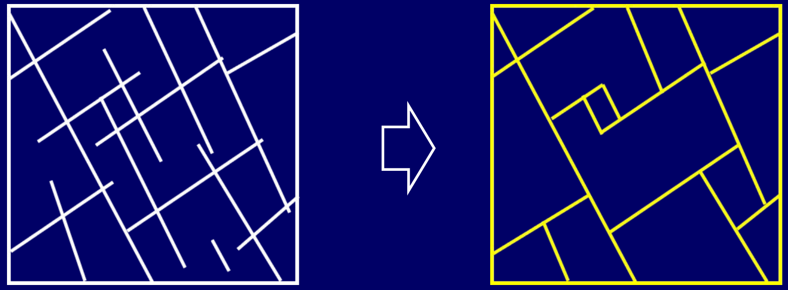
- failure probability $\boldsymbol{P = 1 - e^{-kV\sigma_{cm}^m} \prod_{i=1}^{N} (1 - e^{-\frac{a_{ci}}{\bar{a}_i}})^{\lambda_{vi}V}}$

(4) SMRM permeability model

- Connectivity & permeability tensor



flow between plates



Joint network & connectivity

Joint connectivity $$\eta = \pi \lambda_v \bar{t} (\bar{a} + r)^2 e^{-\frac{3r}{\bar{a}}}$$

Conn. Radius $$r \geq \frac{1}{2\lambda_i \sin\theta}$$

Perm. tensor $$K_{ij} = \frac{g}{12\nu} \sum \bar{t}^2 \eta (\delta_{ij} - n_i n_j)$$

5. Extended applications from 4 SMRM models

(1) Rock Mass structure parameter including: $\lambda_v, RQD, a_m, \eta$

Example: *RQD*

Classical:

$$RQD = (1 + 0.1\lambda)e^{-0.1\lambda}(100\%)$$ **(Sen, 1984)**

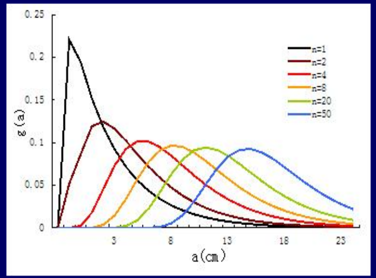
$$RQD = (1 + 0.1\lambda)e^{-0.2\lambda}(100\%)$$ **(Wu, 1993)**

Improved:

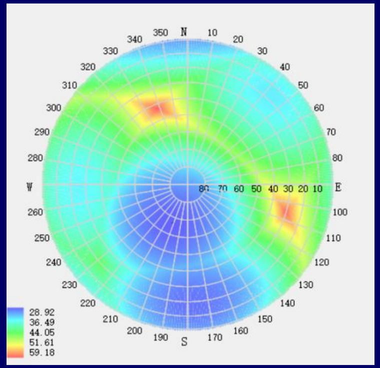
$$RQD = \int_t^L f(x)dx$$

Any L:

$$RQD = \frac{e^{-\lambda t} - e^{-\lambda L}}{1 - e^{-\lambda L}} \rightarrow e^{-0.1\lambda}(100\%)$$



a_m



RQD

(2) Elastic modulus

- Theoretic basis
  - stress-strain model $e_{ij} = C_{ijst}\sigma_{st}$

$$e_{11} = C_{1111}\sigma_{11} \qquad E_m = \frac{\sigma_{11}}{e_{11}} = \frac{1}{C_{1111}}$$

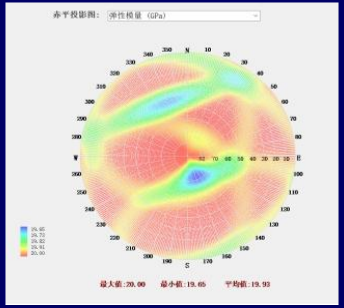
$$C_{1111} = \frac{1}{E}[1 + \alpha \sum_{p=1}^{m} \lambda\bar{a}\,(k^2n_1^2 + \beta h^2))n_1^2]$$

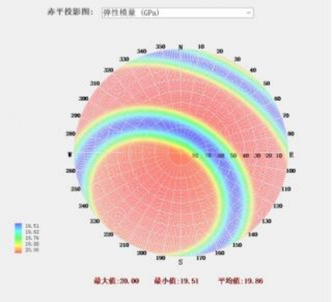
$$E_m = \frac{E}{1 + \alpha \sum \lambda\bar{a}[k^2n_1^2 + \beta h^2(1 - n_1^2)]n_1^2}$$

- Factors affecting E_m: ***m, λ, a, n, k, h***
- Weakening coefficient & Anisotropic index

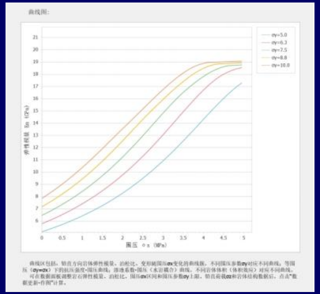
$$\xi_E = \frac{E_{mmin}}{E_{mmax}} \qquad \zeta_E = \frac{E_m}{E}$$

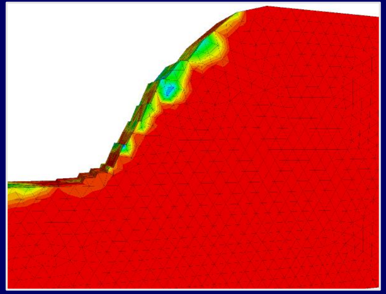
- Vary with direction





- Vary with stress condition





(3) Poisson’ s ratio

- Theoretic basis

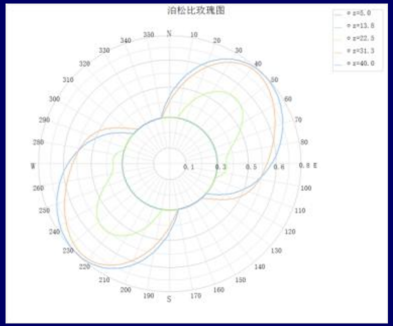
$$e_{ij} = C_{ijst}\sigma_{st}$$

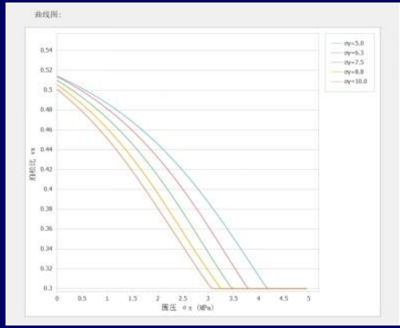
$$\begin{cases} e_{11} = C_{1111}\sigma_{11} \\ e_{x1} = C_{xx11}\sigma_{11} \end{cases} \qquad \nu_{x1} = \frac{e_{x1}}{e_{11}} = -\frac{C_{xx11}}{C_{1111}}$$

$$\begin{cases} C_{1111} = \frac{1}{E}[1 + \alpha \sum_{p=1}^{m} \lambda\bar{a}\,(k^2 n_1^2 + \beta h^2))n_1^2] \\ C_{2211} = \frac{1}{E}[-\nu + \alpha \sum_{p=1}^{m} \lambda\bar{a}\,(k^2 - \beta h^2)n_1^2 n_2^2] \end{cases}$$

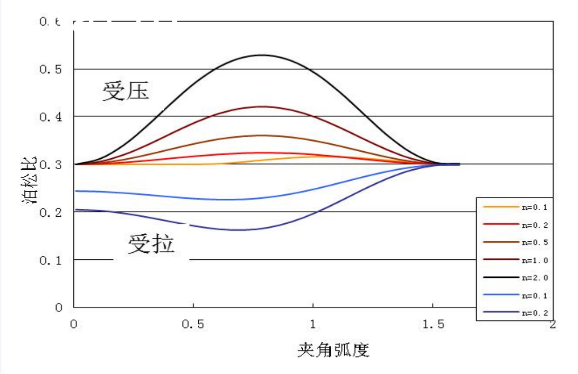
- Anisotropy & larger/less Poisson’ s Ratio Effects

- vary with direction and confining pressure





- Effect of larger/less Poisson’ s



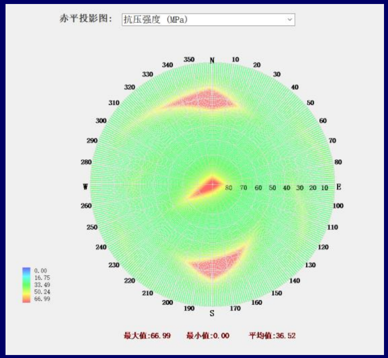
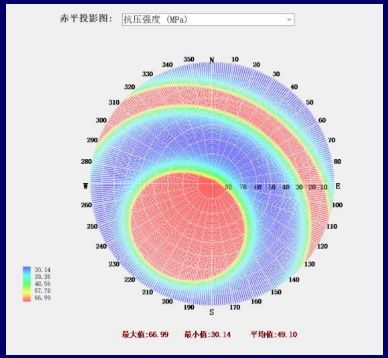
(4) Triaxial compressive strength

- Triaxial compressive strength

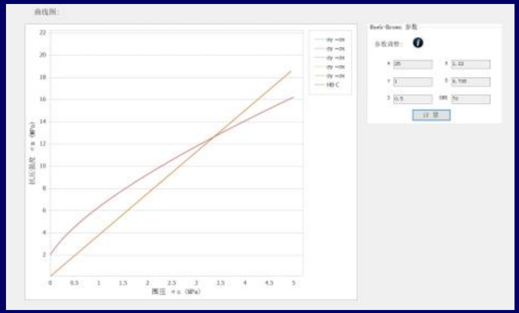
$$\sigma_{1c} = \min(\sigma_{cr},\ \sigma_{cj},\ i = 1,2,\ldots m)$$

$$\begin{cases} \sigma_{cr} = \sigma_3 \tan^2\theta + R_c, \qquad \theta = 45^\circ + \dfrac{\varphi_r}{2} \\ \sigma_{cj} = \dfrac{-B - \sqrt{B^2 - 4AC}}{2A}, \quad \begin{cases} A = [(1+f^2)n_3^2 - 1]n_3^2 \\ B = 2[af + (1+f^2)\sigma]n_3^2 \\ C = a^2 + 2af_j\sigma + (1+f^2)\sigma^2 - p^2 \\ a = \dfrac{K_{Ic}}{2}\left(\dfrac{\pi}{\beta a_m}\right)^{1/2} + c \\ \sigma = n_1^2\sigma_x + n_2^2\sigma_y \\ p^2 = n_1^2\sigma_x^2 + n_2^2\sigma_y^2 \end{cases} \end{cases}$$

- vary with direction

- vary with confining pressure & comparing with Hoek-Brown strength



(5) Shear strength

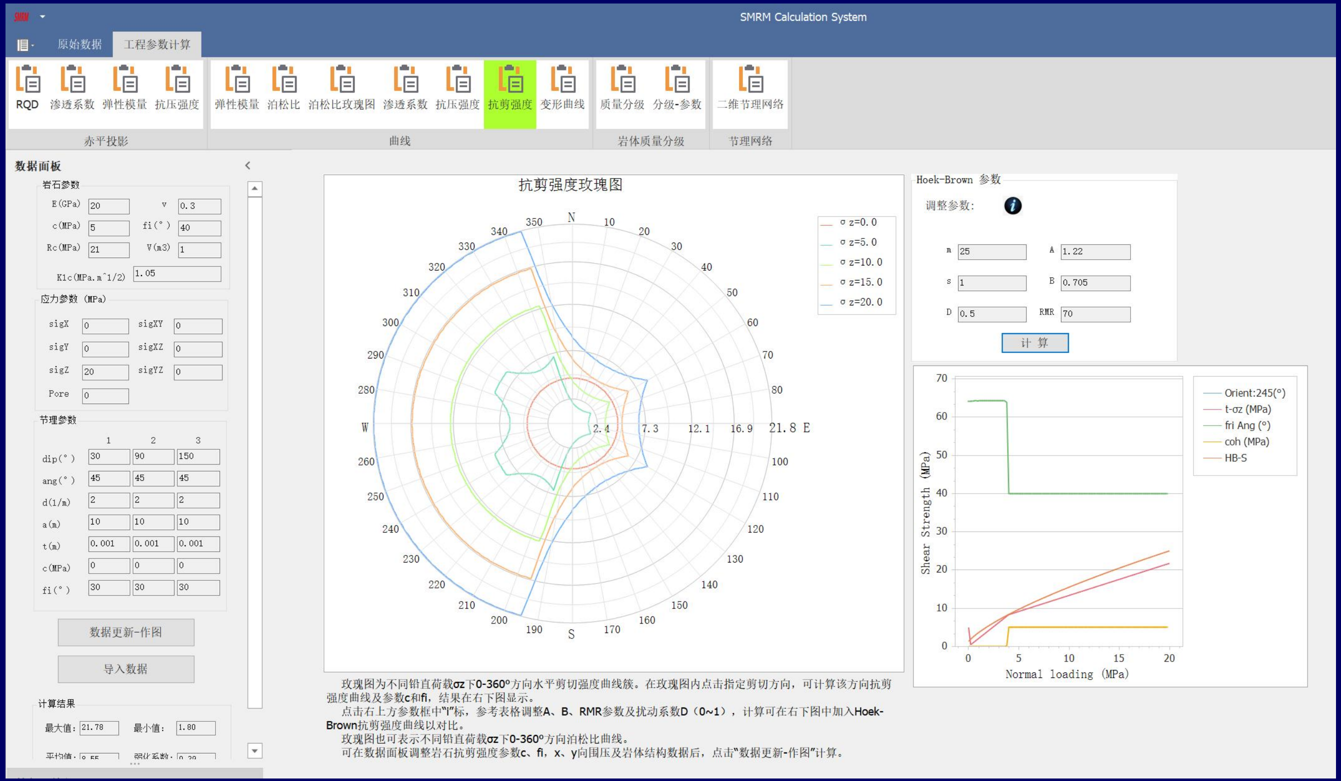
- For Rock Mass

$$t_{xz} = \min(t_{xzr},\ t_{xzji}) \quad \begin{cases} t_{cr} = \sigma\tan\varphi_r + c_r \\ t_{xzj} = -\dfrac{B + \sqrt{B^2 - 4AC}}{2A} \end{cases}$$

- For joints:

$$\begin{cases} \sigma = n_1^2\sigma_x + n_2^2\sigma_y + n_3^2\sigma_z \\ p^2 = n_1^2\sigma_x^2 + n_2^2\sigma_y^2 + n_3^2\sigma_z^2 \end{cases}$$

$$\begin{cases} A = n_1^2 + n_3^2 - 4(1 + f^2)n_1^2 n_3^2 \\ B = 2[\sigma_x + \sigma_z - 2(1 + f^2)\sigma - 2af]n_1 n_3 \\ C = p^2 - \sigma^2 - (a + f\sigma)^2 \\ a = \dfrac{K_{Ic}}{2}\left(\dfrac{\pi}{a_m}\right)^{1/2} + c \end{cases}$$



data panel

SMRM rose curve series

SMRM curve, c, ϕ, Hoek-Brown curve

(6) Permeability Parameter

Permeability Parameter

$$K = \frac{\pi g}{12\nu} \sum \lambda_v \bar{t}_0{}^2 \, \eta(\delta_{ij} - n_i n_j) m_i m_j$$

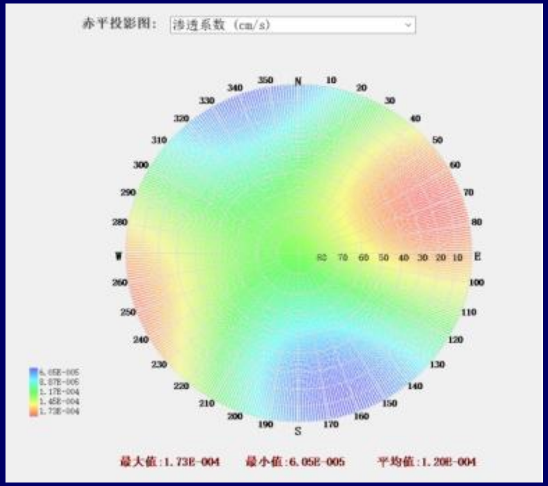
Coupled fluid-stress

$$K = \frac{\pi g}{12\nu} \sum \lambda_v \bar{t}_0{}^2 \, \eta(\delta_{ij} - n_i n_j) m_i m_j e^{-\frac{3\sigma}{k_n}}$$

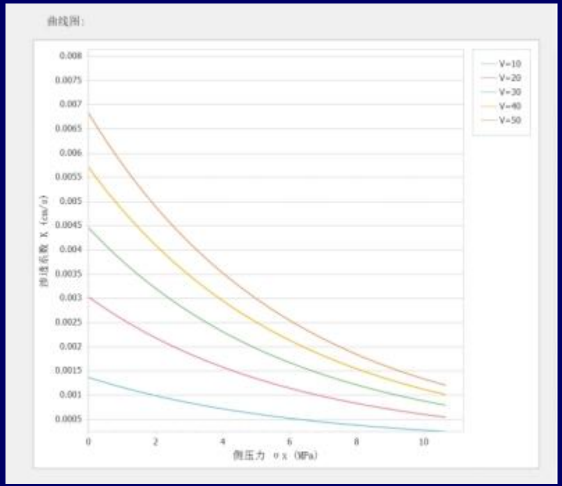
Size effect

$$K = \frac{\pi g}{12\nu} \sum \lambda_v \bar{t}^2 \, \eta(\delta_{ij} - n_i n_j) m_i m_j \ln^3(\lambda_v V)$$

Stereogram projection



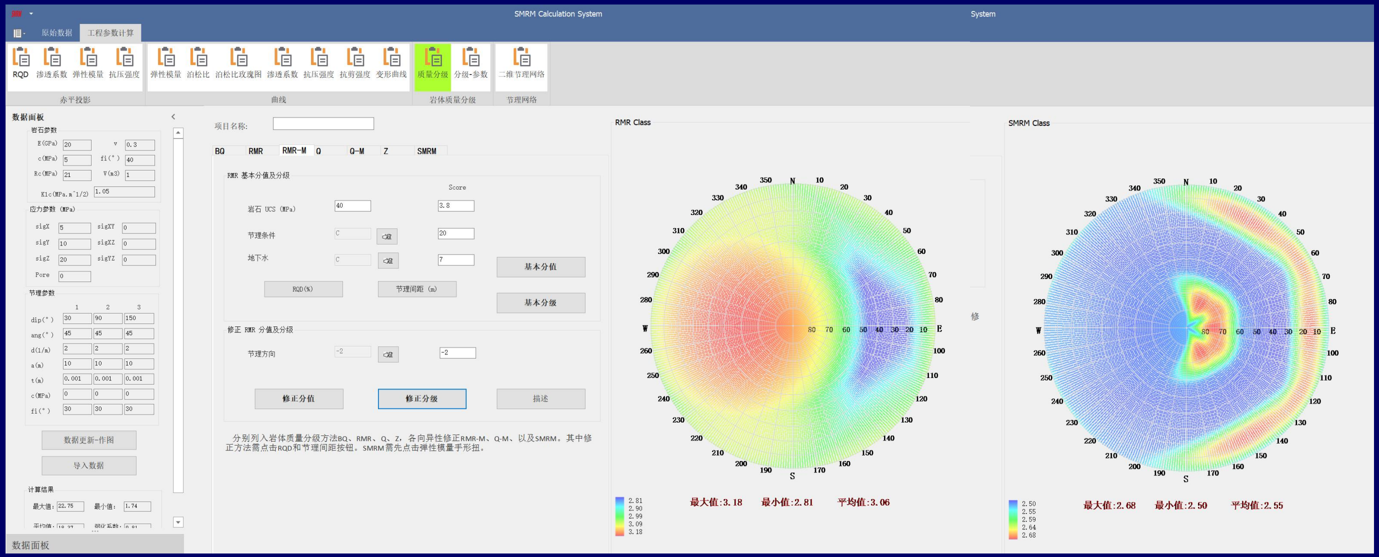
Curves of coupled fluid-stress & size effect



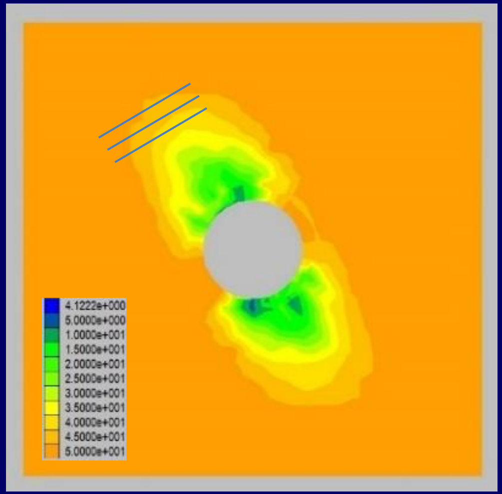
(7) SMRM quality rating & point cloud

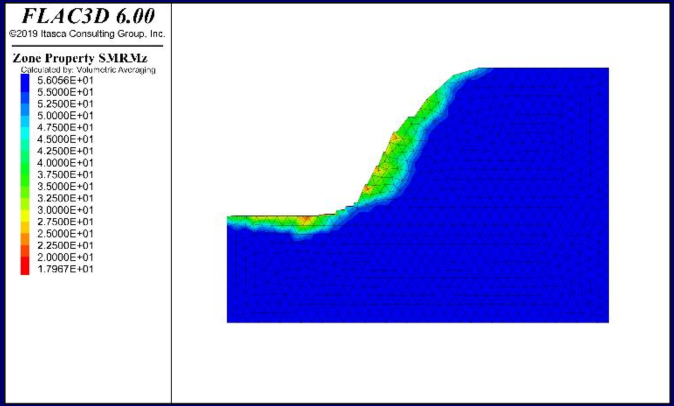
- Deformation based classification

$$SMRM = \begin{cases} \frac{1}{2}(E_m + 100) & E_m > 10\text{GPa} \\ 40\lg E_m + 10 & E_m \leq 10\text{GPa} \end{cases}$$



Mosified RMR system & SMRM system





Point cloud of SMRM rating

- # Case of SMRM classification

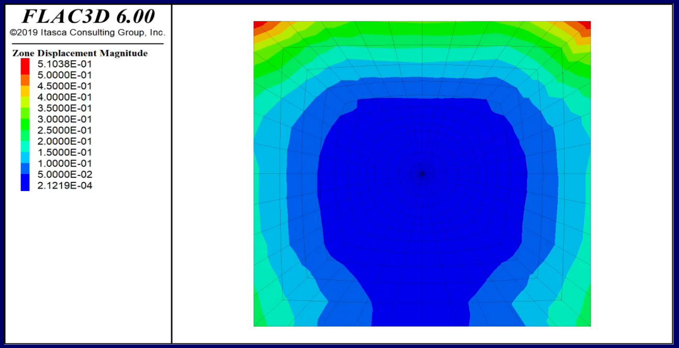
62% change of tunnel linings in a railway due to error of quality rating， leading to investment increase 4 billion RMB.

Carbonaceous slate

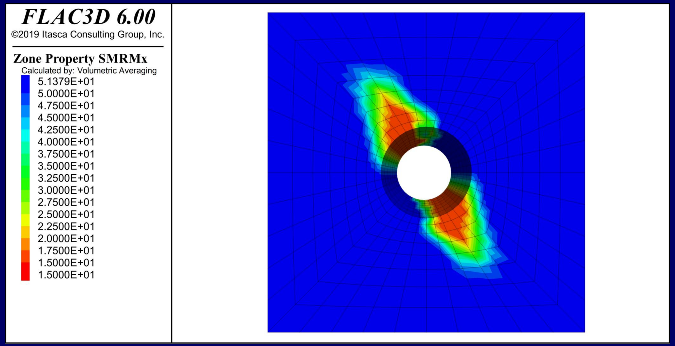
Damage of linings

- Re-evaluate rock mass quality & suggest reinforcement

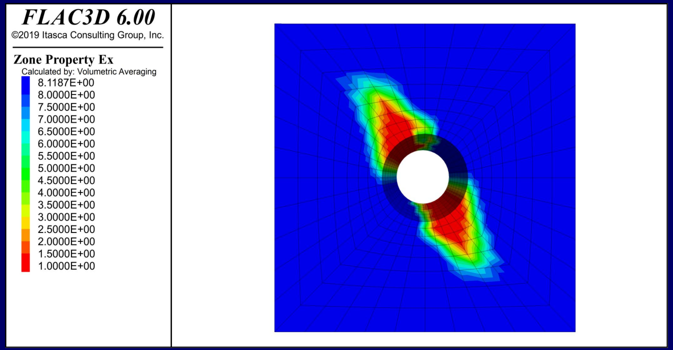
Single inclined bedding, 500m deep, symmetric stress



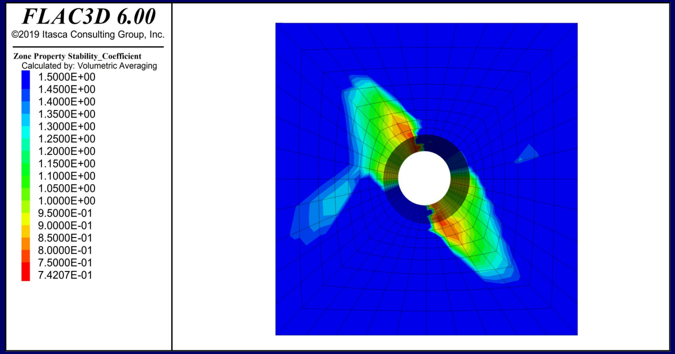
Asymmetric deformation



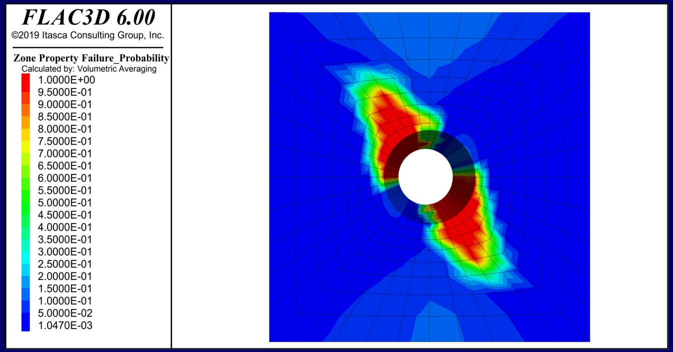
Rock quality rating



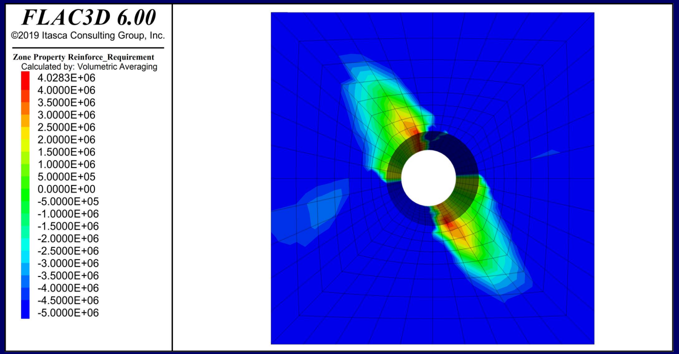
Elastic modulus



Stability coefficient



Failure probability



Reinforcement requirement

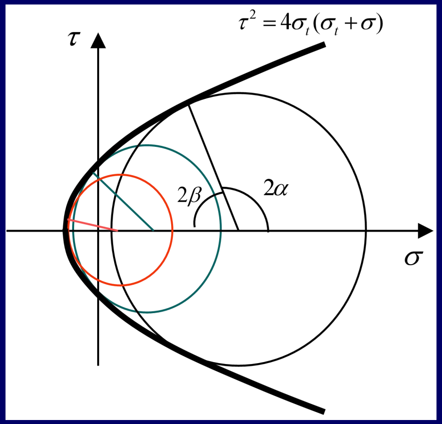
(8) Rock burst & distribution

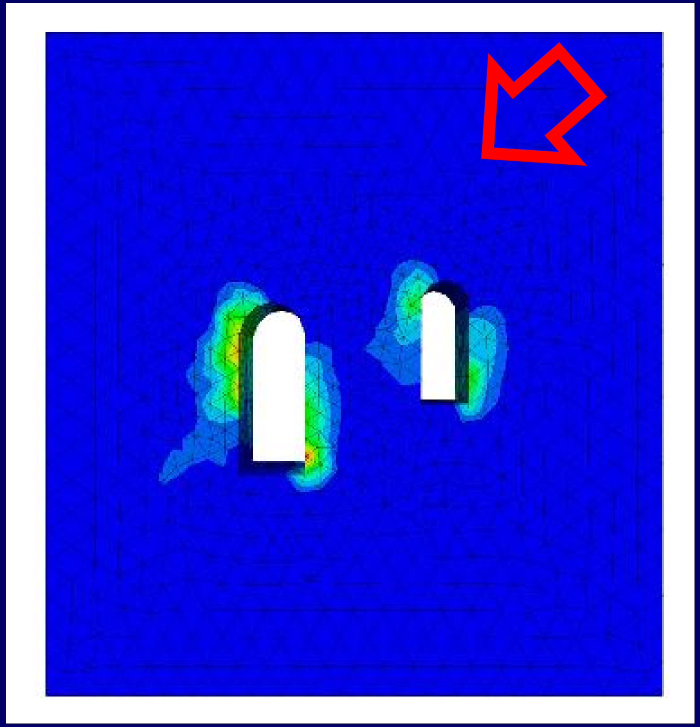
- **Griffith Model for rock burst:**

$$(\sigma_1 - \sigma_3)^2 = \sigma_c(\sigma_1 + \sigma_3)$$

- **Small failure angle:**

$$2\beta = \arccos\frac{\sigma_c}{2(\sigma_1 - \sigma_3)}$$

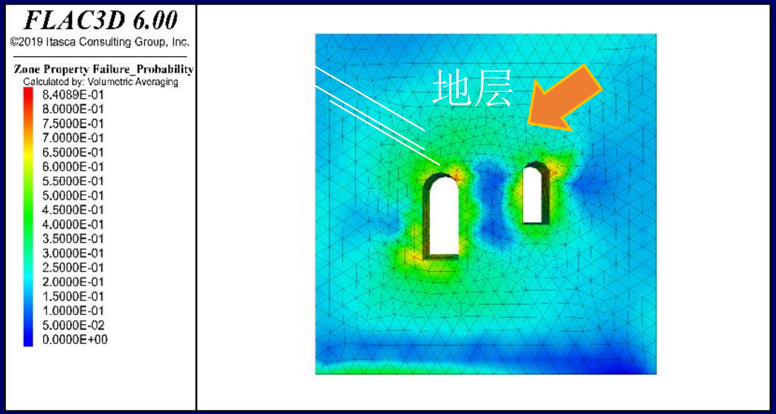




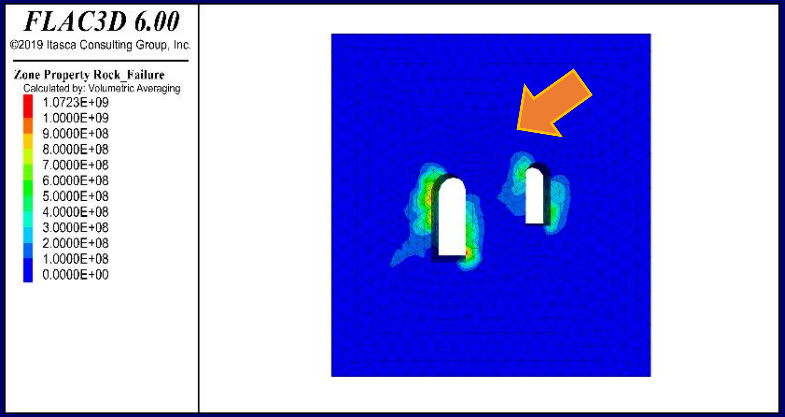
Rock burst distribution

- **Power house of Jinping No.I Hydropower Station**

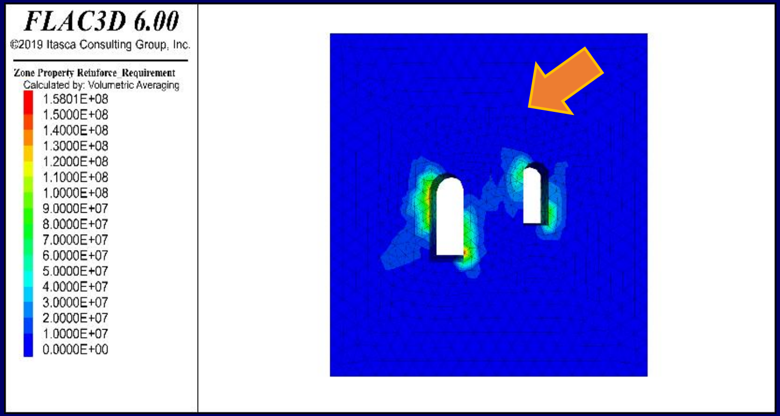
Inclined marble, 200m deep, incline stress



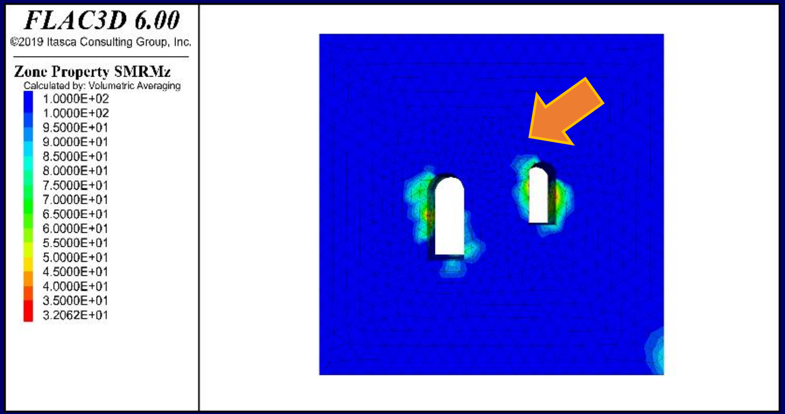
Failure probability



Rock burst



Reinforcement requirement



Rock quality

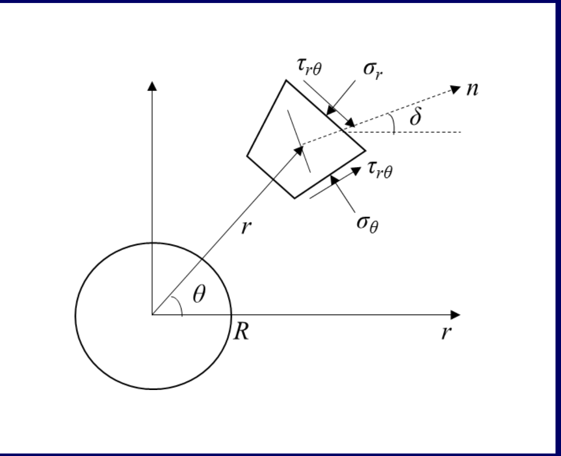
(10) Analytical solution for circular tunnel

displacement:

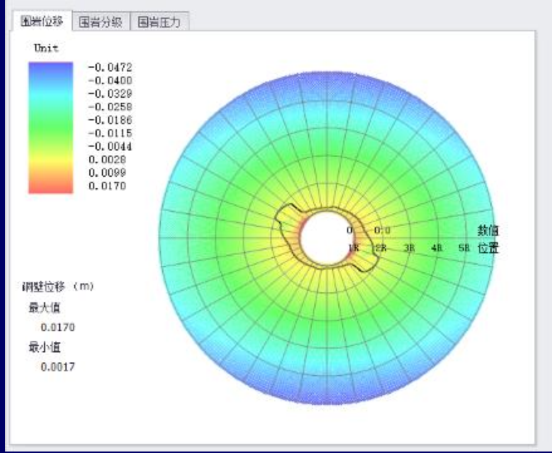
$$u_p = \frac{1}{E'}\{(1 - vT_0 + \frac{8}{\pi}B)\,((r + \frac{R^2}{r})\bar{\sigma} - (r + \frac{4R^2}{r} - \frac{R^4}{r^3})\,\tau\cos 2\theta)$$
$$+ \frac{8}{\pi}C_0(r - \frac{2R^2}{r} + \frac{R^4}{r^3})\tau\sin 2\theta + \frac{8}{\pi}(D_0\bar{\sigma} + F)r - v'R_0 r\} + A$$

Rock pressure:

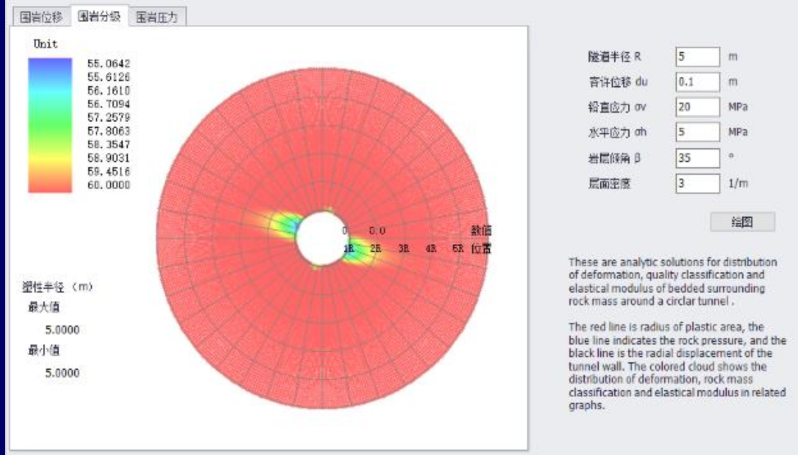
$$p = \frac{u - \Delta u}{\frac{1}{E'}\left(1 - v'T_0 + \frac{8}{\pi}BR\right)}$$



Force diagram



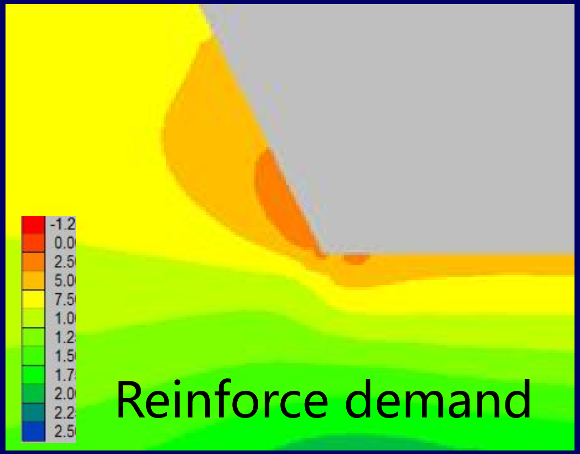
Radial displacement

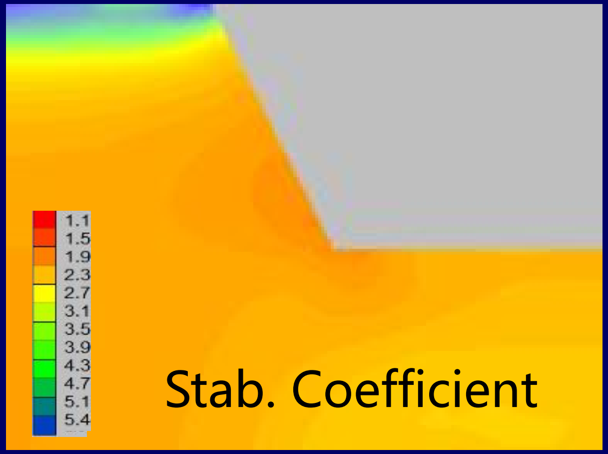


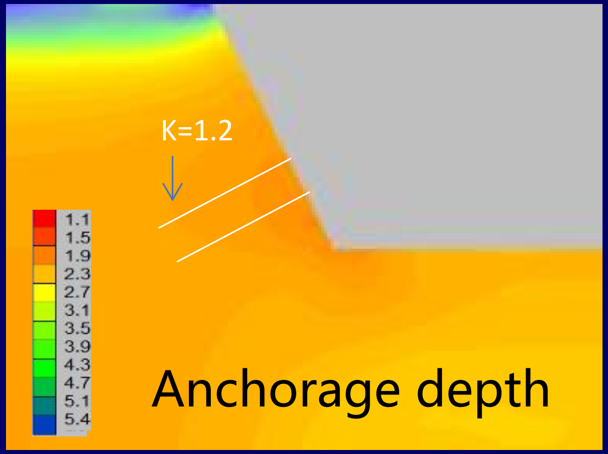
Rock quality distribution

(11) Active reinforcement

- Strength improvement : $\sigma_1 = \sigma_3 \tan^2\theta + \sigma_c$ $\qquad \theta = \frac{\pi}{4} + \frac{\phi}{2}$
- Reinforce demand : $\Delta\sigma_3 = \frac{\sigma_1 - \sigma_c}{\tan^2\theta} - \sigma_3$
- Stability. Coefficient: $K = \frac{\tau_c}{\tau} = \frac{\sigma_1 - \sigma_{3c}}{\sigma_1 - \sigma_3}$
- Unit anchorage force: $F = (\sigma_1 - \sigma_3)(1 - K)\, A$



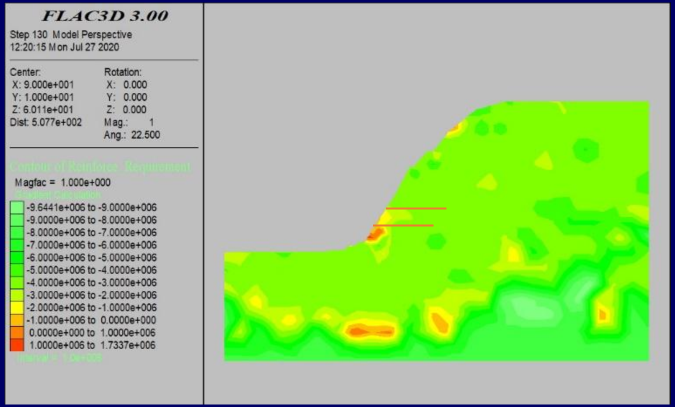




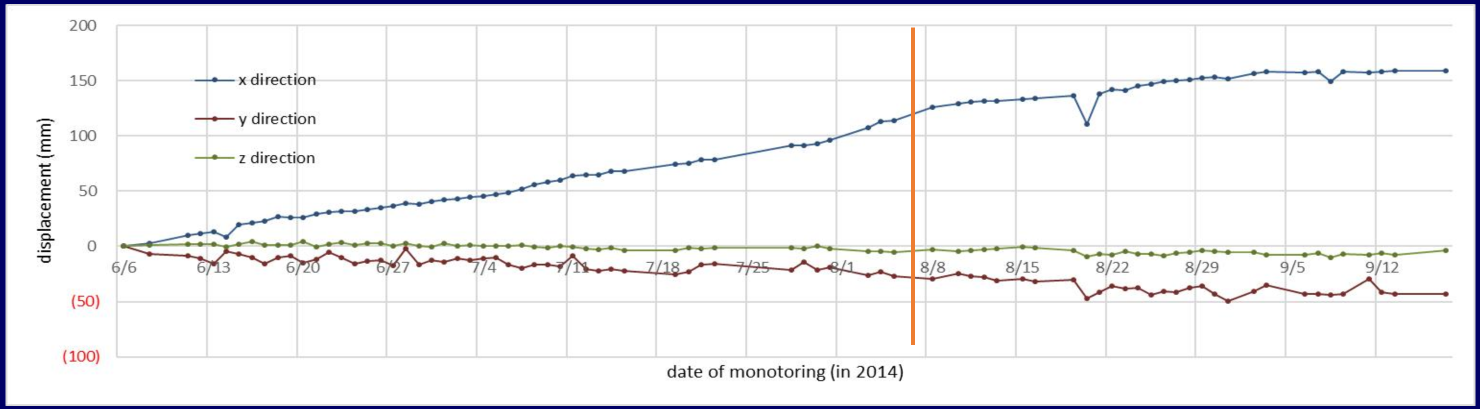
- # Slope, hydropower station, Congo

85m high, sandstone, active reinforcement

excavation



Reinforce demand



displacement

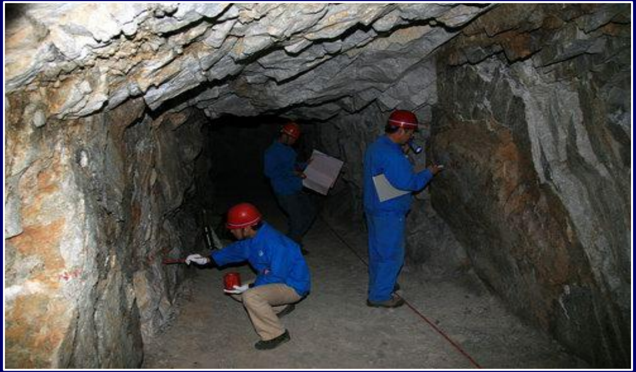
III. APPLIED TECHNIQUE OF SMRM

1. Data acquisition

- Difficulties：hard field work and rock mechanical tests





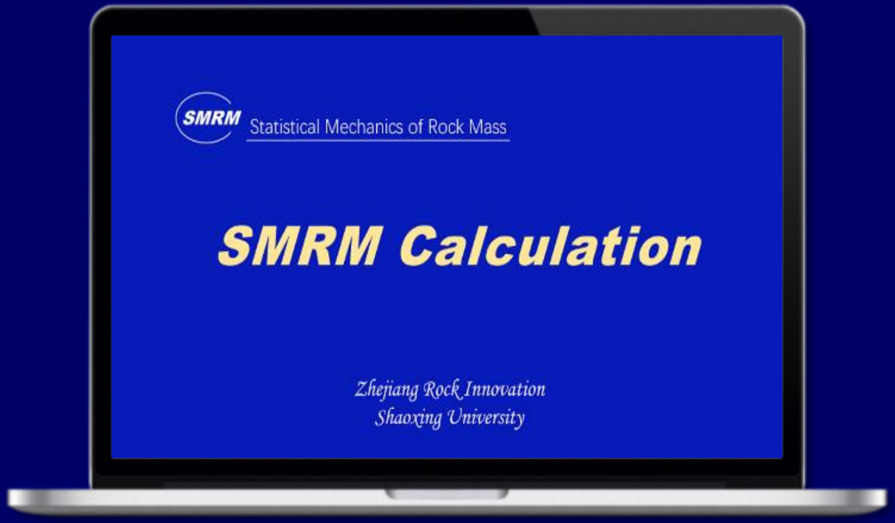



Our solution:

---- SMRM one stop service system

Portable lab

E-Note APP



Calculation platform

Carry the Lab to the site for rock mechanical tests

Portable laboratory

Coring & making samples at the site

Field testing

- The portable Laboratory

Main parts of the laboratory

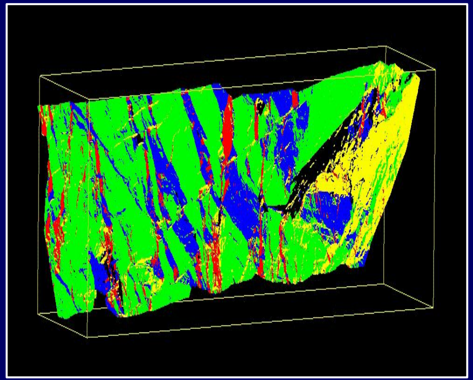
Box-type & Backpack type

- **Interpretation of discontinuity network**

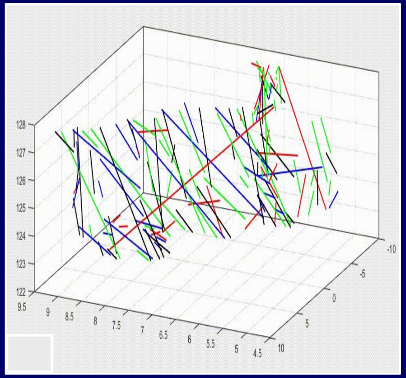
Non-contact measurement

The real rock

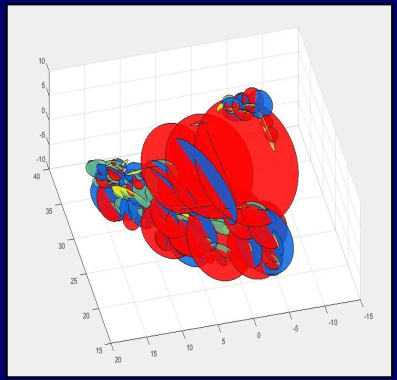
3D point cloud map



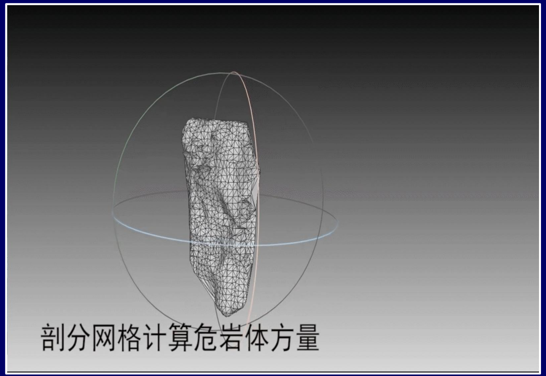
Joint interpretation



Geometric parameter



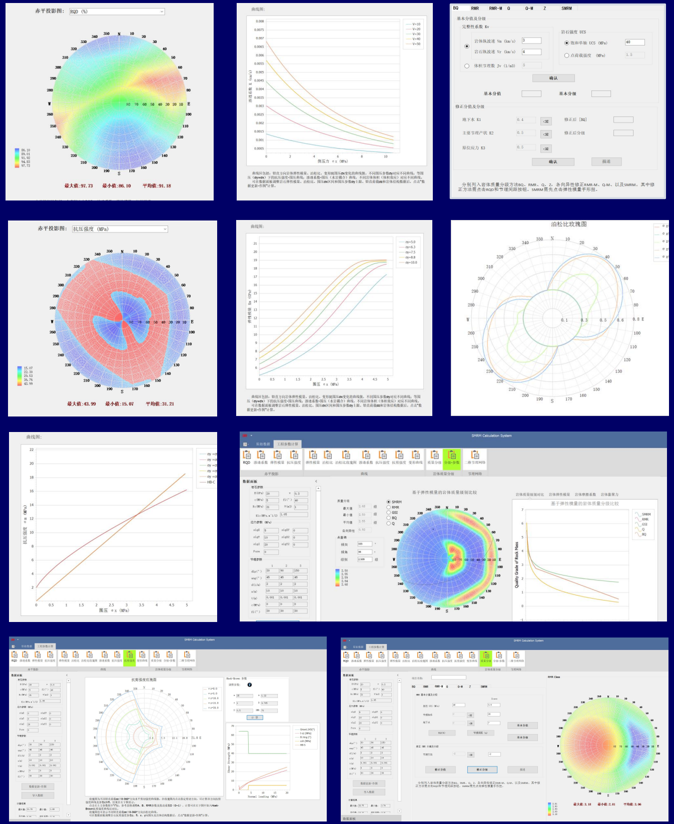
Joint model



Block location & volume

2. “SMRM Calculation” System

The calculation cloud platform



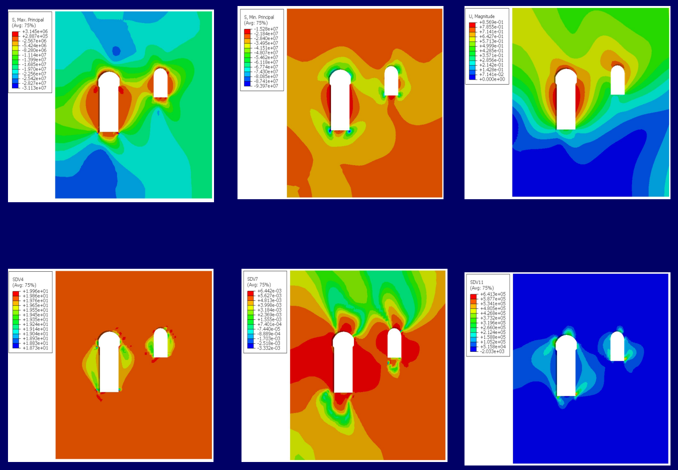
Calculated Parameters

- structural para λ_v, RQD
- elastic modulus E_m
- Poisson’s Ratio v
- compressive strength σ_{1c}
- shear strength t
- permeability $K(\sigma,V)$
- SMRM classification
- rock burst
- tunnel deformation
- active reinforcement

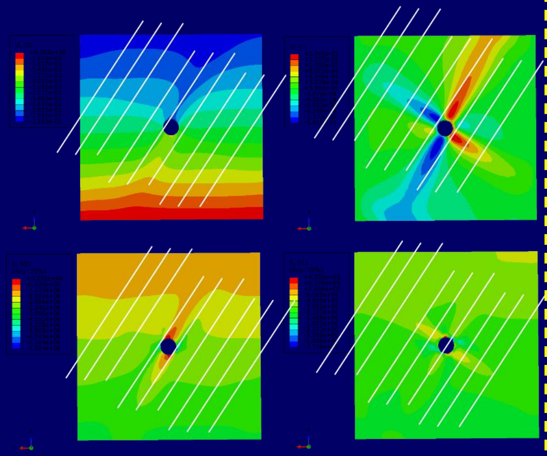
3. Numeric Calculation - modelSMRM module

- Be used to different software platform

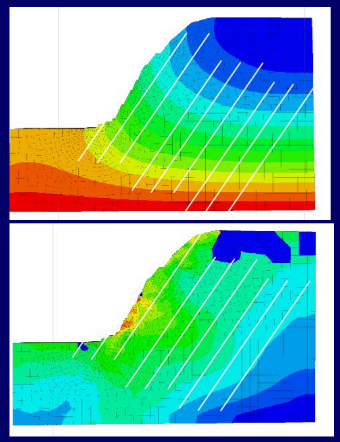
FfsiCAS-SMRM



ABAQUS-SMRM



Flac3D-SMRM



4. Applied cases

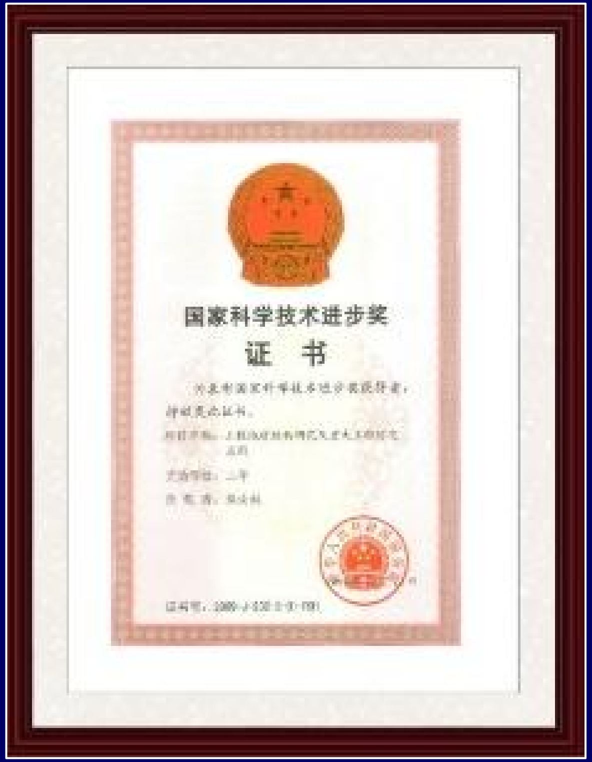
- Slopes：Pubugou, Jinpin I, Xiaowan, Three Gorges, Laoyingshan, Tiantaishan, QBT, Jiangya, Zongo II in Congo
- Tunnels：Jinpin I，Lan-Yu, JiTuHun, Tianping, Badaling








5. Recognition from the society



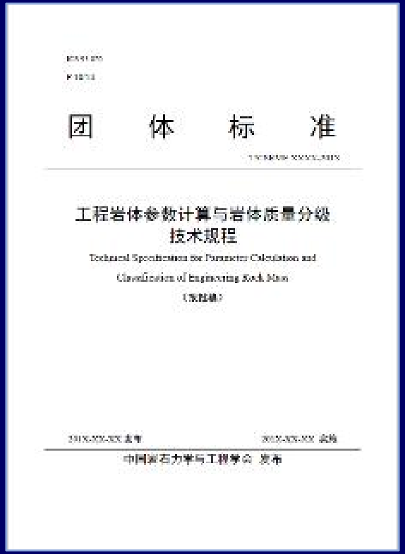
S-T Award, China State Council

Hans Cloos Medal, IAEG

1st Qian Lecture
CSRME

5th Gu Lecture
CEG

Certificate
MHUD, China



Standard
CSRME

SMRM
MOOC

CONCLUSION REMARKS

- Engineering Geology is the basis for rock engineering construction, but needs to be improved in quantification;
- We are devoting to make SMRM to be an engineering geo-mechanics, a new system for fundamental research of scientists, also for a practical tools for rock engineers;
- Application have shown that SMRM is helpful for understanding and solving geological and rock mechanical problems in engineering practice.
- However, we have a long way to go to make SMRM be a science and widely recognized and applied in engineering construction in the future.

THANKS！