Geological Uncertainties Associated with 3-D Subsurface Models

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Abstract

Subsurface models are generally built from both subjective interpretation and mathematical interpolation/extrapolation techniques. These models are therefore uncertain, but their uncertainty is rarely expressed in a geological forecast. In this paper, an evaluation method of geological uncertainties related to 3-D subsurface models is proposed and tested on a real case. This method is based on the subsurface model, which is considered the most probable prediction (best guess). The various geological interfaces are handled as Gaussian random fields to which a model of spatial variability describing possible fluctuations around the best guess is applied. Several structural constraints, such as the shape of folds and thickness of layers are accounted for in the model. At this point, the local variance can be estimated throughout the study area by application of the simple kriging technique. Finally, the variability is converted into probabilities of occurrence of the various rock masses present in the study area. The probabilities are calculated according to intersection rules governing the stratigraphic sequence of the subsurface model. They enable one to probabilistically model subsurface structures in the form of a three-dimensional probability field.

Keywords: Uncertainties; 3-D geological models; Engineering geology

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1. Introduction

Geological models are built from data collected in the field such as boreholes, geophysical measurements, pilot tunnels or geological mapping. However, these data are always limited in number. Subsurface models are thus largely built on subjective interpretation and mathematical interpolation/extrapolation techniques. Such subjective information often leads to uncertainty, which is rarely in a geological forecast. Reliable areas are usually not distinguished from those where the model should be read with caution.

For activities sensitive to geological conditions, uncertainties can lead to significant financial risks or inappropriate strategies. Thus, it is not enough to build the best geological model possible; it is also important to locate and quantify the uncertainties, e.g. to forecast tunnel construction cost and time. For a long time in the field of tunneling, decision-aid tools such as ADCT (Descoeudres et al., 1993) require geological uncertainty as an input. These uncertainties are often poorly evaluated.

A procedure for quantifying geological uncertainties is therefore of invaluable assistance to engineers trying to estimate progress rate and costs for tunneling. Detailed evaluation of uncertainties can also reveal areas that may require further investigation. The aim of this paper is to present a methodology allowing one to estimate uncertainties related to subsurface structures in 3-D geological models. It can be summarized by the three following stages (Figure 1):

1° The basic idea is to start from a best-guessed geological model built up from both observation and interpretation, considered the most probable realization.

2° The subsurface model consists of geological interfaces bounding the various rock types. At this stage, each of these interfaces is treated as a random field. According to the observations available and geological constraints, a variability model is built, describing how reality may differ from the best guess. Random fluctuations are centered around the best guess and the model of spatial variability (a variogram function) expresses how uncertainties increase when one moves away from the observed points.

3° Finally, the variability for each interface is converted by integration into a probability describing the occurrence of the rock types immediately above and below it. These probabilities are updated according to intersection rules governing the stratigraphic sequence of the model. The final result is a set of 3-D probability fields, one for each rock type.

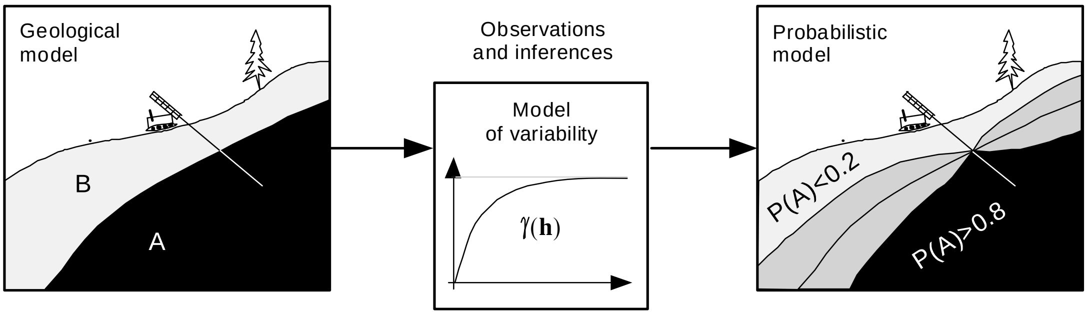


Figure 1. Diagram of methodology.

It is important to note that this method does not provide with simulations (stochastic simulators are presented in detail by Mallet, 2002) but directly with probability fields. It does not depend on the way the best guess was obtained, in particular not on the interpolation method used to build the model. For example, it is compatible with the potentials method, as stated by (Aug, 2004).

2. Assessment of the stochastic model

Subsurface structures are intrinsically deterministic in the sense that they are not the result of a purely random process. However, a full deterministic description is unrealistic in most cases, since an accurate description is unattainable due to various sources of uncertainty.

2.1. The random function concept

Surfaces are the basic elements when modeling structures. They are the geological boundaries of the various rock types. For such features, the variable of interest is the z coordinate of points belonging to the interfaces at each (x, y).

In a three-dimensional study area, three coordinates are necessary to locate points. In order to correctly describe the geometry of stratified or folded structures, we chose to consider a coordinate system, which allows one to take into account the main tectonic directions. This approach is close to the "natural coordinate system" mentioned in Dagbert, David et al. (1984)

or Deutsch and Journel (1998). We thus take into account a coordinate system (u,v,z) that is oriented along the main tectonic directions: u is parallel to the fold axis, v is perpendicular to this axis and z is oriented in a perpendicular direction to the interface (Figure 2).

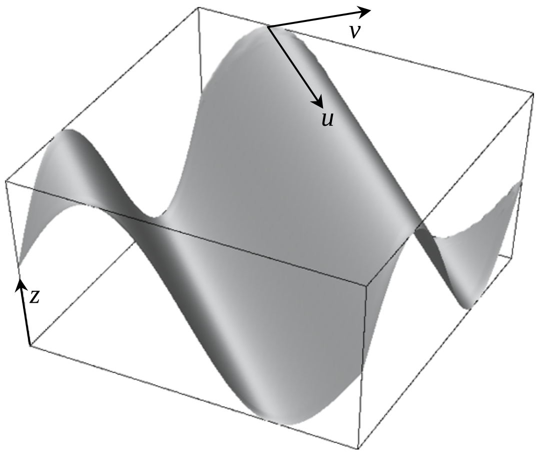


Figure 2. The coordinates u and v are oriented along the main tectonic directions.

For a simple interface, which is not overturned, the coordinate z can be expressed as a function of the first two coordinates u and v. The coordinate z is hence a function of the coordinate vector $\mathbf{u} = (u,v)$, designated $z(\mathbf{u})$.

To correctly describe uncertainties, we should work in a probabilistic framework and consider the study variable z as a random quantity. In a probabilistic framework the available observations are the outcome of a regionalized random process. The various interfaces are handled individually. By considering z as random, the random process $Z(\mathbf{u})$ is by definition a random function:

$$Z(\mathbf{u}) = m(\mathbf{u}) + \sigma(\mathbf{u})\varepsilon(\mathbf{u})$$

This random function is expressed as the sum of a local drift and a local variability. A similar approach can be found in Abrahamsen and Omre (1994). Subsurface models are always a mixed representation of precise information and uncertain predictions; the above stochastic model is also a mixed representation of both deterministic and random information. In our methodology, the geological prognosis is the best guess and uncertainty is added to this deterministic description by introducing variability around it.

- *The deterministic term* $m(\mathbf{u})$

  is the expected value of $Z(\mathbf{u})$ at the location $\mathbf{u}$. It thus represents the most probable outcome of the random process (best guess). It is given by the subsurface model, since the prognosis given by the geologist is assumed to be the best choice among all possible alternatives.

- *The spatially dependent term* $\sigma(\mathbf{u})$

  is a standard deviation which expresses the variability at the location **u**. In a way, it can be regarded as a weighting parameter which provides the amplitude of possible fluctuations of $Z(\mathbf{u})$ at the location **u** according to the spatial distribution of the available information. This parameter will be carefully studied, because it provides a measurement of uncertainties at unsampled locations.

- *The random term* $\varepsilon(\mathbf{u})$

  is a standardized Gaussian random function which expresses erratic fluctuations around the prediction.

2.2. The model of variability

Particular attention should be paid to evaluation of the component expressing the possible dispersion around the best-guessed model. The local standard deviation $\sigma(\mathbf{u})$ - or the local variance $\sigma^2(\mathbf{u})$ - is the key to estimation of uncertainties and should thus be carefully assessed.

As $\sigma^2(\mathbf{u})$ is a spatially dependent term, we should take into consideration the spatial pattern of the phenomenon under study. A rock type is physically continuous and elements near one another might thus be expected to have properties that are somewhat similar. On the average, the smaller the separation between elements of a rock type, the more similar the properties (Einstein and Baecher, 1992). We can thus expect that as the distance increases, confidence in the geological prognosis will decrease. In other words, we can expect that the local variance $\sigma^2(\mathbf{u})$ of the random function $Z(\mathbf{u})$ is small around known locations and that it increases when the distance from an observation increases.

A measure of the spatial variability is given by the semi-variogram $\gamma(\mathbf{h})$. The semi-variogram expresses the variability as a function of the distance between known locations. The separation distance between two sampled locations is measured here as the path length between two samples on the interface. A remarkable property of the variogram is that it can be viewed as an estimation variance. Very often, in practice, the semi-variogram stops increasing beyond a certain distance and becomes more or less stable around a limit value. Under the strong stationarity assumption, this threshold value is simply the *a priori* variance of the random function (Journel and Huijbregts, 1978). For large separation distances, the variogram is, by definition, the variance of the data set.

Although it is occasionally possible to estimate the variogram function through experimental data, it must usually be done on the basis of the knowledge we have on the phenomena as suggested by Abrahamsen and Omre (1994). The variogram must have a physical meaning and parameters must be related to the geology. It seems more judicious to use an empirical variogram model. This empirical model should be chosen according to the spatial pattern of the structures under study, rather than by modeling it with the few available observations. From a statistical point of view, an experimental variogram is certainly more meaningful, but when the behavior of a specific variable really matters, an empirical variogram could be more representative of the underlying process. An experimental variogram will not show the characteristics of the entire process, but only those underlying the observations available in the limited sampling area.

In nature, e.g. when rocks are distorted, it often happens that a wide variety of patterns is produced. Among those, folds are perhaps the most common tectonic structure. They can be found in a wide range of scales and shapes. Fold shape does not have a great impact on its variogram: Sinusoidal fold shape, box fold shape or chevron-like folds produce almost the same variogram. A Gaussian model fits this variogram. The most important parameters are the amplitude and wavelength of folds (Figure 3) (Pomian-Srzednicki, 2001). In fact, the sill value of the variogram is closely related to folds amplitude and the range is related to wavelength.

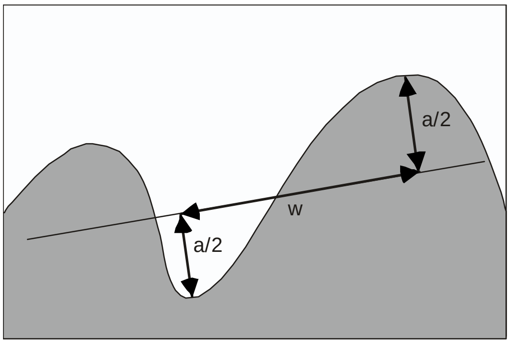


Figure 3. Measurement of the half amplitudes a and wavelength ω of a folded interface.
Sill value is $c \cong a^2/2$ and range is $r \cong 1/\omega$.

Most natural phenomena are not isotropic. For example, folded surfaces present two distinct anisotropic directions along which variability is fairly different. Thus, a 2-D model of variability in a coordinate system oriented according to the main directions of anisotropy is considered (Figure 2). Along the fold axis, small fluctuations are generally observed. The largest variability is observed perpendicular to the fold axis.

2.3. Model extension

In the previous section, a variability model is proposed for each geological interface independently. The way in which each interface interacts with the others must now be considered.

The most constraining relation between the various interfaces is the constant thickness clause. If a given layer has a strictly constant thickness (the layer is said to be isopach), any information concerning one boundary can be transposed to the other boundary of this layer (Figure 4). Also, the way in which the thickness of a layer varies around a fold depends on the fold style. Two types of folds may be taken into consideration:

Parallel Folds

In a parallel fold, the layer thickness measured orthogonally across the layer is constant throughout the fold. With such folds, information can be transposed along a direction perpendicular to the bedding surfaces (Figure 4).

Similar folds.

Similar folds are formed by simple heterogeneous shear deformations in a direction parallel to the axial surface. In similar folds, the layer thickness measured anywhere in the fold in a direction parallel to the axial surface is constant. With such folds, information will be transposed along the shear direction.

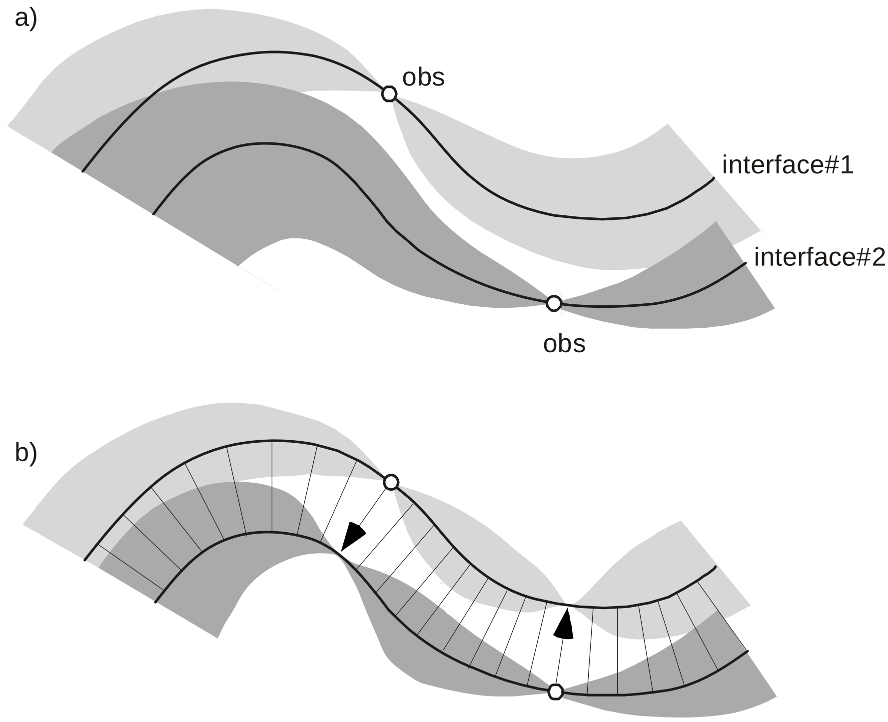


Figure 4. (a) Two interfaces delimiting a parallel-folded layer. Variability is assessed independently for each interface according to the available observations on each interface. (b) If the layer has a constant thickness, information available on one interface can be transposed to the other.

This classification is only based on the measure of constant thickness and does not represent the whole variety of the folds' geometry. However, these two types of folds are the most remarkable ones and any other shape has characteristics ranging between these two models.

The local variance is only estimated according to observations available on the various interfaces and any observation which is not located on an interface will not be taken into account in the variability model. This can lead to inconsistencies, especially when an observation is located near an interface (Figure 5). In order to correct this, we have to locally reduce the variability so that possible fluctuations around the best guess are not overlapping the available observation.

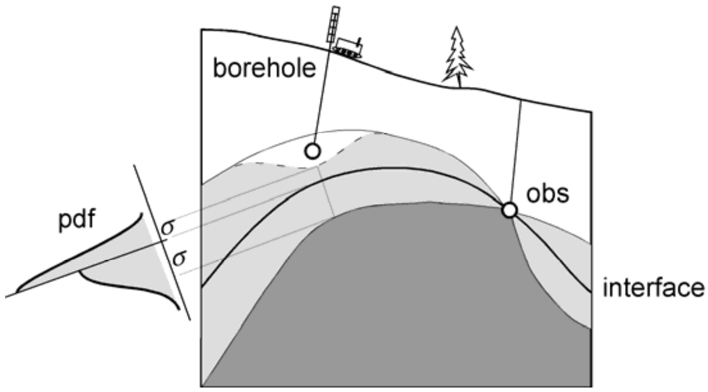


Figure 5. A borehole reaches the vicinity of an interface of the subsurface model. The initial variability model (full line) is not consistent with observations collected along the borehole. In this case, the variance is reduced in order to obtain a corrected variability model (dotted line).

2.4. Estimating the local variance

The key to the description of these uncertainties is the estimation of the local standard deviation $\sigma_Z(\mathbf{u})$ - or the local variance $\sigma_Z^2(\mathbf{u})$. At this point, the kriging technique can enable one to estimate this parameter. Kriging not only provides with an estimation of unknown values; it also provides with the estimation error, often referred to as the kriging variance. Among the many available kriging techniques, simple kriging (SK) is the most adequate in this case, because the mean $m(\mathbf{u})$ is known as it is given by the geological model.

Equality between the kriging variance and the sample variance only exists if there is no evidence of a proportionally effect. In our case, the variance only depends on the distance to observations. Although uncertainties often increase with depth, this is due to the fact that available information is mainly clustered near the surface. There is no relation between the value of the variable and its variance. No proportionally effect should therefore be expected.

3. Assessment of the probabilistic model

In this study, probabilities describe the chances of encountering a certain type of rock at a given location. The probability of the event "rock type *A* is found at $z(\mathbf{u})$" is written P(*A*). It describes the occurrence of rock type *A* at the location $z(\mathbf{u})$.

Probabilities are obtained by the cumulative distribution function $\mathrm{F}(\mathbf{u};\, z)$ of the probability density function $f(z)$. The occurrence of rock type *A* is calculated with the following integral:

$$P(A) = \mathrm{F}(\mathbf{u};\ z) = \int_A f(z)dz$$

As $Z(\mathbf{u})$ is assumed to be Gaussian, the probability density function can be defined by a Gaussian function with $E\{Z(\mathbf{u})\} = m(\mathbf{u})$, the predicted position of the boundary, and $Var\{Z(\mathbf{u})\} = \sigma_{\mathrm{SK}}^2(\mathbf{u})$, the estimated local variance.

Probabilities describing the occurrence of the various rock types can be calculated for any location of the study area as a three-dimensional probability field. However, the results can be more easily read when probabilities are represented with uncertainty bounds. These bounds are defined by a confidence interval of probability and the standard deviation is usually a commonly chosen threshold value:

$$z_{\text{upper bound}}(\mathbf{u}) = m(\mathbf{u}) + \sigma(\mathbf{u}) \quad \text{and} \quad z_{\text{lower bound}}(\mathbf{u}) = m(\mathbf{u}) - \sigma(\mathbf{u})$$

The width of the confidence interval is a direct measure of uncertainties and can therefore indicate areas where the predicted location of the interface is reliable and areas where it is uncertain.

4. Combining uncertainties according to stratigraphic relations

When subsurface structures are composed of two or more interfaces, the geological model is built by organizing and combining these surfaces altogether according to several stratigraphic relations. Two main types of stratigraphic relation are taken into account:

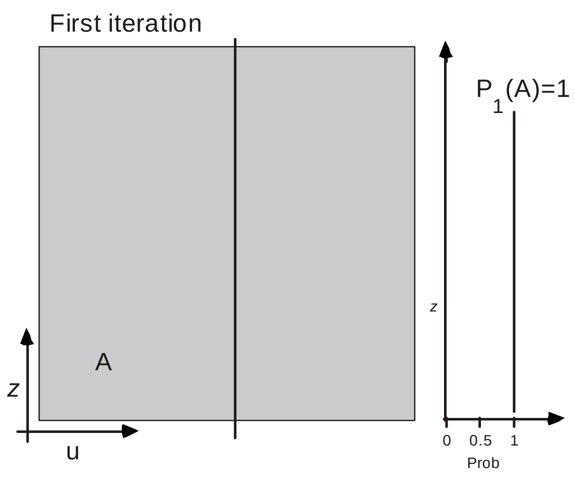
- depositional surfaces
- erosion surfaces.

By organizing and combining depositional and erosion surfaces into a specific sequence, it is possible to build any kind of geological structure, from the simplest to the most complex (Mayoraz, 1993). The same process of intersecting the interfaces will be considered to calculate the probabilities.

4.1. Depositional surfaces

A sequence of deposited layers is formed by piling layers one on top of the other and, in the same way, probabilities are first calculated starting from the bottom to the top by adding successive layers to the model. As we rise in the sequence, the probabilities are updated in order to take into account the presence of the new layer. This is an iterative process, issued from the Bayes theorem, as used by Tacher and Parriaux (1997). The example below (Figure 6 a to c) shows three iterations of this process:

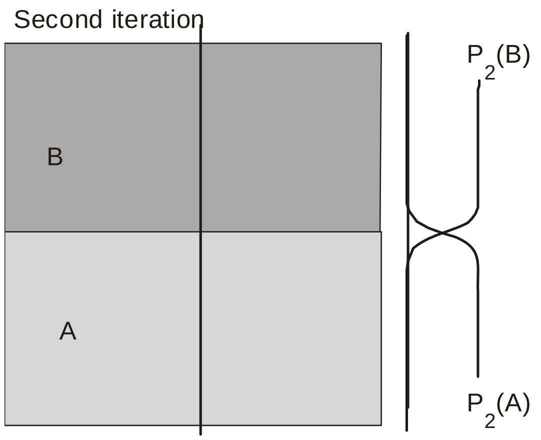
a)



<u>First iteration</u> ($N = 1$)

$P_1(A) = 1$

b)



Second iteration ($N = 2$)

Starting from the bottom of the model, the first interface separates the layer A from the layer B.

$F_1(\mathbf{u};z)$ is the probability density function (pdf) to encounter the first interface along u = **u**.

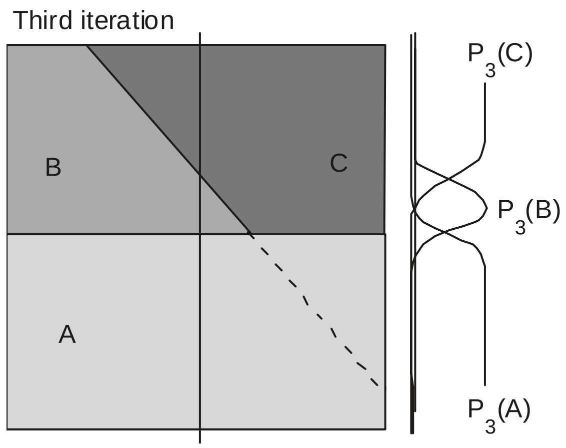
$$P(B) = F_1(\mathbf{u}; z)$$

The probability of finding layer A is updated according to the presence of the new layer B.

$$P_2(A) = P_1(A)\,[1 - P(B)]$$

$$P_2(B) = P(B)$$

c)



Third iteration ($N = 3$)

A second interface is added to the model. This interface is the upper boundary of layer B.

$$P(C) = F_2(\mathbf{u}; z)$$

The probability of finding the layer B is updated according to the presence of the new layer C.

$$P_3(A) = P_2(A)$$

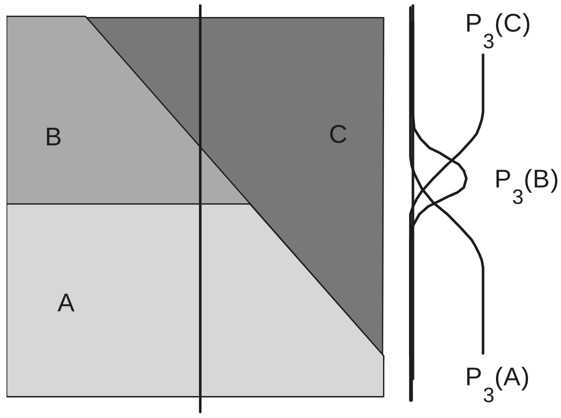
$$P_3(B) = P_2(B)[1 - P(C)]$$

$$P_3(C) = P(C)[1 - P_2(A)]$$

Figure 6 a to c. Three iterations illustrating the updating process when successively introducing depositional layers in the geological model. On the right, the resulting probabilities along the vertical profile (vertical line in the model). Since the B-C interface is depositional, it is interrupted by layer A; thus P3(A) is unchanged: P3(A) = P2(A)

4.2. Erosion surfaces

An erosion surface is introduced when a discontinuity appears in the stratigraphic sequence. Typically, there are two geological features that can be represented as erosion surfaces: faults and discordance surfaces. Erosion surfaces are singular objects and thus cannot be handled in the same way as depositional interfaces. The example below (Figure 7) shows how an erosion surface is treated:



Erosion surface

The layer C lies in unconformity and its lower boundary, the second interface, is an erosion surface.

$$P(C) = \mathrm{F}_2(\mathbf{u}; z)$$

The probability if finding layers A and B is updated according to the presence of the new layer C.

$$P_3(A) = P_2(A)[1 - P(C)]$$

$$P_3(B) = P_2(B)[1 - P(C)]$$

$$P_3(C) = P(C)$$

Figure 7. Updating of pdf if layer C lies in unconformity. Since the B-C interface is erosional, it interrupts layer A; thus, P3(A) must be updated.

5. Application to a real case - Lötschberg base tunnel, Switzerland

5.1. Introduction

The Lötschberg base tunnel is one of two planned tunnels which are part of the new north-south railway project through Switzerland. The two giant constructions, scheduled to be completed in 2012, constitute the most important civil engineering project ever undertaken in Switzerland.

The Lötschberg base tunnel will be about 35 km long, crossing the entire Lötschberg massif. Despite generally favorable conditions, particular zones may cause technical problems during construction. The Triassic evaporite rocks of the southern portal are one of the most problematic formations. In this area, the structural prediction is critical along the planned tunnel route, but uncertainties about subsurface structures are important for such a deep construction (Dudt and Descoeudres, 1999). This area represents a good case study for testing the potential usefulness of our methodology.

5.2. The 3-D subsurface model

The 3-D subsurface model of the southern portal was built according to the few available sources of information: 5 cross sections (Dolivo, 1982), 4 boreholes made during the preliminary geological survey for the tunnel construction (Ziegler, 1997), structural and tectonic observations (Dolivo, 1982) and a detailed 10’000 geological map (Dolivo, E. Nouvelles observations structurales au SW du massif de l'Aar entre Visp et Gampel., unpublished). The resulting 3-D model of this area is presented in Figure 8.

5.3. Calculation of uncertainties

The subsurface model is considered the best guess. In the study area, folds are nearly cylindrical and their axes gently plunging in the SE direction (Dolivo, 1982). A directional variability model was therefore set up. Along the fold axes, the Gaussian variogram’s range is set to 1’500 m, which is the minimum half-wavelenght in this direction. Perpendicular to fold axes, the range is set to 500 m. In both directions, the variogram's sill is set to 20'000 m^2 because the maximum variability around the expected position of the interfaces is assumed to be SQR(20'000) ≈ 140 m. The local variance was then estimated around each interface of the subsurface model by the simple kriging technique according to available observations (outcrops and boreholes) and the following anisotropic semi-variogram:

$$\gamma(\mathbf{h}) = \gamma(h_{//}, h_{\perp}) = 20'000. \left[1 - \exp\left(-\sqrt{\left(\frac{h_{//}}{1'500}\right)^2 + \left(\frac{h_{\perp}}{500}\right)^2} \right) \right]$$

Finally, probabilities of occurrence were calculated for each rock mass according to the stratigraphic sequence of the subsurface model. The resulting probabilities are shown as a 3-D probability field (Figure 9).

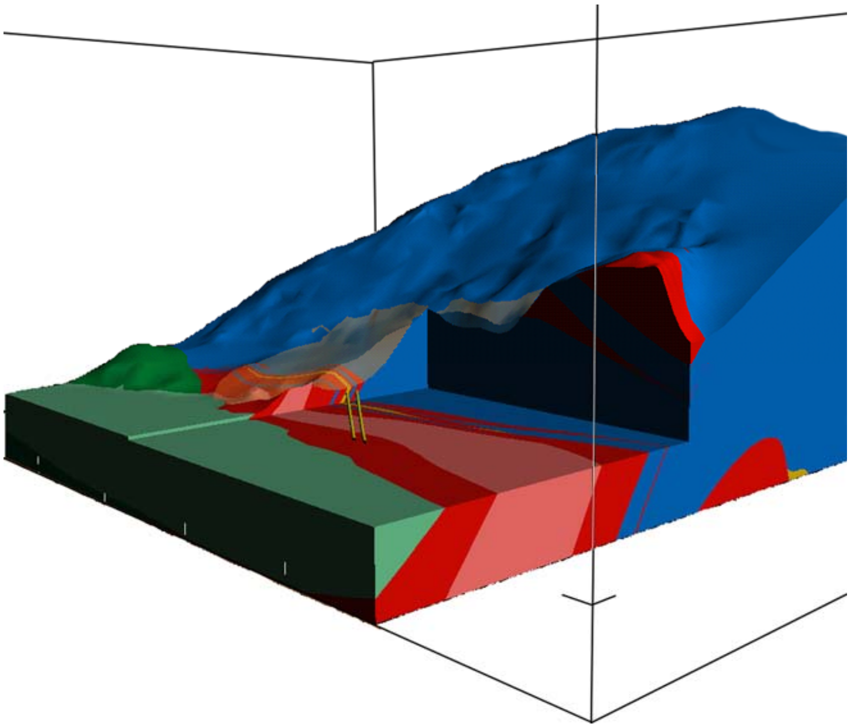


Figure 8. A view in the north-west direction of the 3-D subsurface model in the area of the southern portal created with the EarthVision® modeling software. The two drives of the Lötschberg tunnel are represented by tubes.

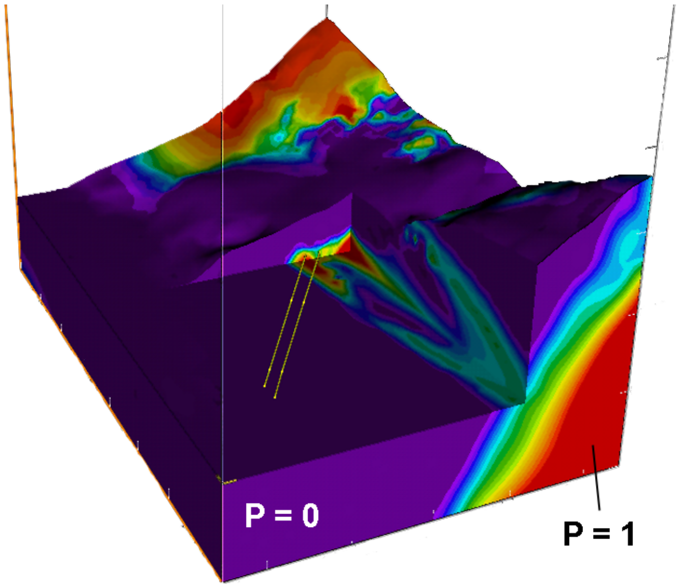


Figure 9. A view in the north-west direction of the resulting 3-D probability field in the area near the southern portal representing the occurrence of Liassic rocks.

Such a representation shows the overall probable occurrence of a rock type in the study area. This 3-D representation has the advantage of providing a rapid understanding of the resulting uncertainties in the whole study area.

Probabilities can also be obtained along the tunnel track itself in order to focus results on the area of interest (Figure 10). Such result can then be directly integrated into a decision-aid system in order to evaluate geological risks for tunnel construction. A one-dimensional profile makes it possible to represent all probabilities along the tunnel drive on the same figure. Thus, probabilities can be confronted in order to compare the relative probabilities of encountering the various rock masses.

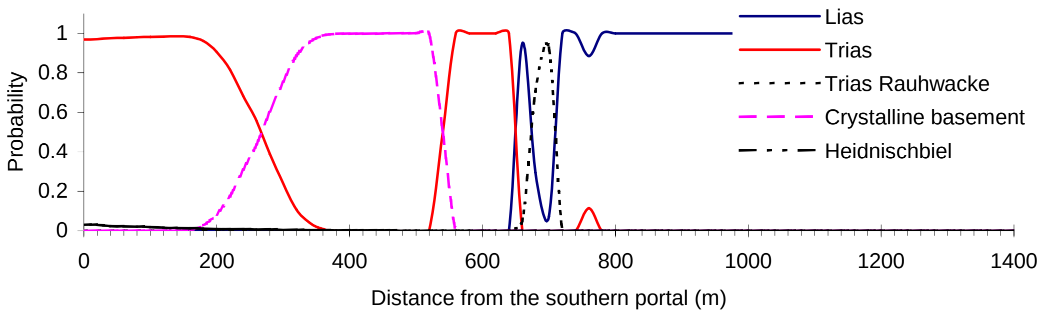


Figure 10. Profile of probabilities along the Lötschberg tunnel drive near the southern portal. All rocks of the subsurface model are present in this profile.

In this last profile, it is interesting to compare the occurrence of the various rock masses. Uncertain predictions can be revealed when probabilities are smoothly varying from one place to another but, on the other hand, when the probability profile is steep, few uncertainties are expected.

6. Conclusion

This paper presents a methodology for the estimation of uncertainties related to geological structures in 3-D subsurface models according to several geological constraints. These constraints, such as fold shapes, constant thickness and stratigraphic relations, can provide a detailed description of uncertainties, which is closer to the natural phenomenon under investigation than the one obtained by usual geostatistical methods. Prior guess is also accounted for, since uncertainties are evaluated according to the best-guessed geological model. At each location of the study area, this methodology allows us to evaluate the chances of finding a certain rock type in a full three-dimensional context. This evaluation is expressed in terms of probabilities, which are a direct quantification of uncertainties.

This methodology can successfully help engineers and geologists in the evaluation of uncertainties over a whole 3-D study area. It can be useful for identifying areas of greater uncertainties, which can be targeted for further data collection, thus optimizing the site characterization. This methodology can also serve as an accurate evaluation of geological risks for tunnel construction, since the resulting probabilities can easily be integrated into an existing decision-aid system.

At this point, it must be realized that, just as there cannot be prediction without a prior model, there cannot be assessment of uncertainty of that prediction without a prior model. As our model of uncertainties is centered on the best-guessed geological model, it is like all models arbitrary in essence. One should thus be aware that the methodology does not attempt to provide an estimation of global uncertainties which account for wrong choices or omissions the geologist might have made when building up his model. In areas where structures are expected to be complex, the model of variability can be adapted in order to describe such uncertainties.

Several aspects of this research are to be developed further. Additional deterministic information, such as dip measurements, can be taken into account in future uncertainty models. Also, soft information issued from geophysics must be considered by adding a nugget effect to the variogram model. Soft kriging and indicator kriging (e.g. Goovaerts, 1997) remain to be considered to introduce semi-quantitative information in the local variance estimation.

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